TILING RECTANGLES

UNIVERSITY OF FLORIDA MATH CIRCLE

1. What is the minimal number of squares needed to tile a 2x5 rectangle?

2. What is the minimal number of squares needed to tile a 4x5 rectangle? 4x6? 4x7? 5x6? 6x7? How are you sure?

3. In general, what is the minimal number of squares needed to tile an mxn rectangle, where m and n are integers?[1]

4. Let $f(m,n)$ represent the minimal number of squares needed to tile an mxn rectangle. It might be good to fix one of the variables and see what happens to the other one. For example, consider the first 10 cases where one of the dimensions is fixed, like $f(2, n)$. Can you find the missing numbers below? What patterns do you see? Are there any patterns you can express as formulas?[2] Can you prove it?[3]

$f(2,1) =$ ____
$f(2,2) =$ ____
$f(2,3) =$ ____
$f(2,4) =$ ____
$f(2,5) =$ ____
$f(2,6) =$ ____
$f(2,7) =$ ____
$f(2,8) =$ ____
$f(2,9) =$ ____
$f(2,10) =$ ____
$f(2, n) =$ ____

5. Can we do something similar for $f(3, n)$? $f(4, n)$? Any other dimensions?

$f(3,1) =$ ____
$f(3,2) =$ ____
$f(3,3) =$ ____
$f(3,4) =$ ____
$f(3,5) =$ ____
$f(3,6) =$ ____
$f(3,7) =$ ____
$f(3,8) =$ ____
$f(3,9) =$ ____
$f(3,10) =$ ____
$f(3, n) =$ ____

$f(4,1) =$ ____
$f(4,2) =$ ____
$f(4,3) =$ ____
$f(4,4) =$ ____
$f(4,5) =$ ____
$f(4,6) =$ ____
$f(4,7) =$ ____
$f(4,8) =$ ____
$f(4,9) =$ ____
$f(4,10) =$ ____
$f(4, n) =$ ____

$f(5,1) =$ ____
$f(5,2) =$ ____
$f(5,3) =$ ____
$f(5,4) =$ ____
$f(5,5) =$ ____
$f(5,6) =$ ____
$f(5,7) =$ ____
$f(5,8) =$ ____
$f(5,9) =$ ____
$f(5,10) =$ ____
$f(5, n) =$ ____

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[1] This is an open problem. An open problem is one for which a solution is not yet known. We can still play with it and see where it takes us. Perhaps one of you will solve this problem 10 or 20 years from now!
[2] I encourage you to visit the Online Encyclopedia of Integer Sequences ® at oeis.org to enter sequences you find.
[3] Visit cyfemat.org/open for a document showing the collaborative effort of different people working on this problem. It also has references and links. The images on that document were created using virtual-graph-paper.com.